This week, we need some Group Theory. The minimum goal is to know Theorem 1.2 and Proposition 1.3 in the book in detail. When the book mentions "semigroups," these are spaces of finite words where there may be no inverse. If you have time, I would like to see Theorem 4.2 in the book presented.

Again, I would like to see your proofs and hear your ideas ideas on Thursday. We can discuss any questions you have, too. It is OK to come with lots of ideas and questions, but no finished proofs. Try to get as far as you can in the time you have.

0.1 Preliminaries

With any luck, this will be a lot of review, and it will not take you too long.

- 1. Convince yourself that the collection of distance-preserving maps (isometries) is a group.
- 2. Make sure that you are familiar with the definitions for
 - Abelian and non-abelian groups
 - Group homomorphisms
 - Group action
 - Fixed points under a group action
 - Orbit of a subset under a group action
- 3. Convince yourself that a group, G, acts on itself by left multiplication (sometimes called left translation).
- 4. Look up the definition of
 - a free group (usually the term free semigroup is defined somewhere close by, and this might be nice to know, too).
 - the generators of a group.
 - relations and defining relations upon the generators for a group.

If you have any questions on these, please bring them on Thursday. These are the necessary terms to understand. We will be very interested in non-abelian free groups on two generators. It may be very helpful to write out or draw what that such a group looks like (see Figure 4.1 in the book). Also, feel free to look up videos online that might help with this.

0.2 Things to Present

- 1. At minimum, I would like to see you present proofs of Theorem 1.2 and Proposition 1.3 from the book. The proofs are fairly short. I would also like you to draw the pictures of that the proof is doing. That is, find a way of drawing the non-abelian free group on two generators, and figure out which sets in the proof correspond to which sets in your picture. Then figure out what the steps of the proof are doing to those sets in the picture.
- 2. If you have time this week, I would like you to read Theorem 4.2 in the book. The proof is a bit longer, but make sure you read and understand Theorem 1.2 first.
- 3. If you still have time this week, I would like you to present Theorem 4.2 in the book.