0.1 The Hausdorff Paradox

This week, we will see the Hausdorff Paradox! This is the last paradox we will consider before the full Banach-Tarski paradox. The proof is not spelled out so much in the book, because we already have all the ideas for it. The first thing to do is to put it all together. To do so, please read:

1. Theorem 1.10 in the book. Understanding this theorem is very, very important. It is the foundation of all the paradoxes to come. It may be helpful to read some of the discussion before and after Theorem 1.10 in the book, just to provide context.

2. Theorem 2.3. The proof for this is in the discussion before the actual theorem. If begins at the paragraph, "Each element of the free group of rotations..."

0.2 To Present

I would like you to present,

1. The full proof of Theorem 2.3. This has generally two parts: the construction mentioned before the statement of Theorem 2.3, and Theorem 1.10. I want to see all the details of the construction and all the details of the proof of Theorem 1.10.

The discussion in Chapter 2 is very good, though for our purposes, Theorem 2.1 and Theorem 2.2 are not so important to know in detail. Theorem 2.1 just gives two rotations in $\mathbb{R}^3$ which generate a non-abelian free subgroup in $S0_3$ (the group of all rotations in $\mathbb{R}^3$). But I encourage you to read the surrounding discussion. If you want to use two explicit rotations in your presentation of Theorem 2.3, feel free to skim Theorem 2.1 and use those.

0.3 If you have time

Please begin reading Chapter 3. We will come back to Chapter 2, later, but begin reading Chapter 3 from the beginning up to but not including Proposition 3.4 . More specifically, I want you to read all the discussion and to know Definition 3.1 and Definition 3.3. Theorem 3.2 is interesting, but you do not need to know the proof. Stop when you get to 3.4.

Again, please just do your best. You have been doing an amazing job so far, but I don't have a good sense of how much work 4 hours is for you. So feel free to let me know if it is either too much or too little. We can adjust easily :)